

# Exclusive weak $B$ decays involving $\tau$ lepton in the relativistic quark model

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Semileptonic and leptonic  $B$  decays are analyzed in the framework of the relativistic quark model. Special attention is paid to the decays involving  $\tau$  lepton. It is found that the calculated particular decay branching fractions are consistent with available experimental data within error bars. However, the predicted and recently measured ratios  $R(D^{(*)})$  of the  $B \rightarrow D^{(*)}\tau\nu_\tau$  and  $B \rightarrow D^{(*)}l\nu_l$  branching fractions differ by  $1.75\sigma$  for  $R(D)$  and by  $2.4\sigma$  for  $R(D^*)$ .

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Recently significant progress has been achieved in studying the exclusive semileptonic and weak leptonic  $B$  decays. Branching fractions of these decays involving  $\tau$  lepton were measured [1–6]. It was found that the experimental values of the semileptonic  $B \rightarrow D^{(*)}\tau\nu_\tau$  [1–4] and leptonic  $B \rightarrow \tau\nu_\tau$  [4–6] decays differ by more than  $2\sigma$  from the theoretical predictions in the framework of the Standard Model. This deviation is mostly pronounced when theoretical expectations are compared with the recent BaBar data for the ratios  $R(D^{(*)})$  of the semileptonic  $B \rightarrow D^{(*)}\tau\nu_\tau$  and  $B \rightarrow D^{(*)}l\nu_l$  decay branching fractions [3]. In such ratios most of the uncertainties, e.g. the ones emerging from the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element  $V_{cb}$ , cancel. However, such ratios of decay rates still depend on meson form factors, and thus are model dependent, but this dependence is significantly milder than for the individual branching fractions. These measurements initiated discussion in the literature on the possible New Physics contributions to the decay processes involving  $\tau$  lepton (see e.g. [7] and references therein).

In this letter we give predictions for these weak semileptonic and leptonic  $B$  decays in the framework of the relativistic quark model. The model is based on the quasipotential approach in quantum field theory with the QCD motivated interaction. Hadrons are considered as the bound states of constituent quarks and are described by the single-time wave functions satisfying the three-dimensional Schrödinger-like equation, which is relativistically invariant [8]:

$$\left( \frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R} \right) \Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_M(\mathbf{q}), \quad (1)$$

where the relativistic reduced mass is

$$\mu_R = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \quad (2)$$

$M$  is the meson mass,  $m_{1,2}$  are the quark masses, and  $\mathbf{p}$  is their relative momentum. In the center of mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}. \quad (3)$$

TABLE I: Calculated ( $|V_{cb}| = 0.039 \pm 0.0015$ ) and measured branching fractions of semileptonic  $B \rightarrow D^{(*)}l\nu_l$  and  $B \rightarrow D^{(*)}\tau\nu_\tau$  decays (in %).

Decay	Theory	Experiment [4]	Experiment [3]
$B^0 \rightarrow D^- l^+ \nu_l$	$2.14 \pm 0.16$	$2.18 \pm 0.12$	$2.23 \pm 0.11 \pm 0.11$
$B^0 \rightarrow D^- \tau^+ \nu_\tau$	$0.68 \pm 0.05$	$1.1 \pm 0.4$	$1.01 \pm 0.18 \pm 0.12$
$B^+ \rightarrow D^0 l^+ \nu_l$	$2.32 \pm 0.17$	$2.26 \pm 0.11$	$2.31 \pm 0.08 \pm 0.09$
$B^+ \rightarrow D^0 \tau^+ \nu_\tau$	$0.73 \pm 0.05$	$0.77 \pm 0.25$	$0.99 \pm 0.19 \pm 0.13$
$B^0 \rightarrow D^{*-} l^+ \nu_l$	$5.51 \pm 0.42$	$4.95 \pm 0.11$	$4.72 \pm 0.05 \pm 0.34$
$B^0 \rightarrow D^{*-} \tau^+ \nu_\tau$	$1.43 \pm 0.11$	$1.5 \pm 0.5$	$1.74 \pm 0.19 \pm 0.12$
$B^+ \rightarrow D^{*0} l^+ \nu_l$	$6.00 \pm 0.46$	$5.70 \pm 0.19$	$5.40 \pm 0.02 \pm 0.21$
$B^+ \rightarrow D^{*0} \tau^+ \nu_\tau$	$1.56 \pm 0.12$	$2.04 \pm 0.30$	$1.71 \pm 0.17 \pm 0.13$

The interaction quasipotential  $V(\mathbf{p}, \mathbf{q}; M)$  consists of the perturbative one-gluon exchange part and the nonperturbative confining part [8]. The Lorentz structure of the latter part includes the scalar and vector linearly rising interactions. The long-range vector vertex contains the Pauli term (anomalous chromomagnetic quark moment) which enables vanishing of the spin-dependent chromomagnetic interaction in accord with the flux tube model.

The semileptonic  $B$  decay form factors were calculated in our model in Ref. [9]. For the heavy-to-heavy  $B \rightarrow D^{(*)}$  transitions the heavy quark expansion has been employed. Leading and subleading order Isgur-Wise functions were explicitly determined as overlap integrals of the meson wave functions and on this basis the decay form factors were calculated in the whole kinematical range up to  $1/m_Q$  terms. These form factors and decay branching fractions were found to agree well with the available experimental data. We calculated decay rates involving light leptons  $l = \mu, \nu$  since at that time only such decays were measured experimentally. Now we apply these form factors for the consideration of the  $B \rightarrow D^{(*)}\tau\nu_\tau$  decays. The obtained branching fractions for them as well as updated predictions for  $B \rightarrow D^{(*)}l\nu_l$  decays are given in Table I in comparison with experimental data. Both experimental averages from PDG [4] and recent BaBar data [3] are given. For theoretical estimates we use the CKM matrix element  $|V_{cb}| = 0.039 \pm 0.0015$ , which is obtained from the comparison of our theoretical predictions [9] for the products  $F_{D^{(*)}}(w)|V_{cb}|$  and for the  $B \rightarrow D^{(*)}l\nu_l$  decay branching fractions with updated experimental data.<sup>1</sup> The uncertainties in our theoretical predictions originate mainly from the  $|V_{cb}|$  value and higher order  $1/m_Q$  contributions to form factors. We find that almost all measured and calculated values of branching fractions agree within uncertainties. This is also true for the decays involving  $\tau$  lepton. In Fig. 1 we show our predictions for the differential decay rates of the  $B \rightarrow D^{(*)}l\nu_l$  and  $B \rightarrow D^{(*)}\tau\nu_\tau$  semileptonic decays.

Form factors of the heavy-to-light semileptonic  $B$  decays, such as  $B \rightarrow \pi(\rho)l\nu_l$ , have been calculated in our model in Ref. [9] without application of the heavy quark expansion. They were expressed through the overlap integrals of the meson wave functions which are known from the meson mass spectra calculations. The momentum transfer dependence of these form factors was determined in the whole kinematical range without any ad hoc parametrizations. Relativistic effects were systematically taken into account, including negative-energy con-

<sup>1</sup> This value of  $|V_{cb}|$  is in accord with its recent evaluation by the Heavy Flavor Averaging Group [10].

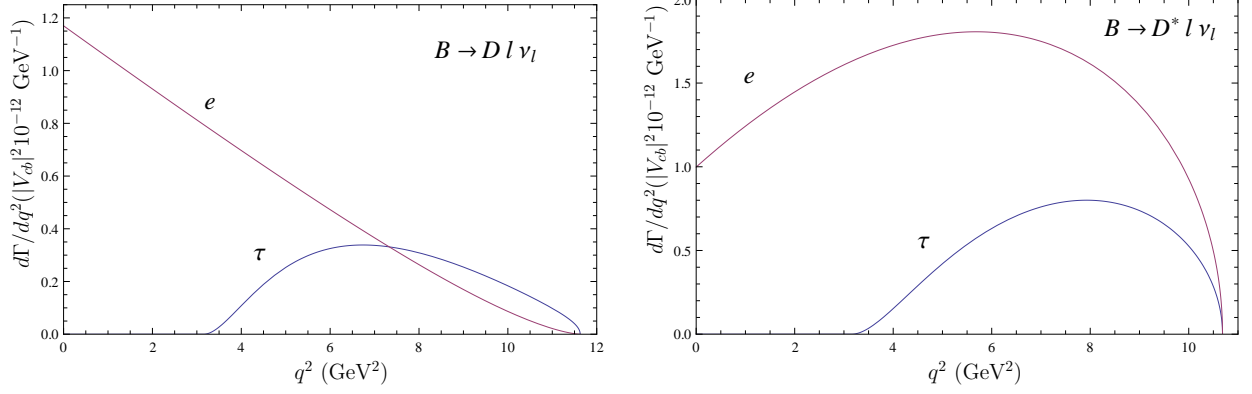


FIG. 1: Predictions for the differential decay rates of the  $B \rightarrow D^{(*)}$  semileptonic decays.

TABLE II: Calculated  $[|V_{ub}| = (4.05 \pm 0.20) \times 10^{-3}]$  and measured branching fractions of the semileptonic  $B \rightarrow \pi(\rho)l\nu_l$  and  $B \rightarrow \pi(\rho)\tau\nu_\tau$  decays ( $\times 10^{-4}$ ).

Decay	Theory	Experiment [4]
$B^0 \rightarrow \pi^- l^+ \nu_l$	$1.37 \pm 0.13$	$1.44 \pm 0.05$
$B^0 \rightarrow \pi^- \tau^+ \nu_\tau$	$0.86 \pm 0.08$	
$B^+ \rightarrow \pi^0 l^+ \nu_l$	$0.74 \pm 0.07$	$0.778 \pm 0.028$
$B^+ \rightarrow \pi^0 \tau^+ \nu_\tau$	$0.47 \pm 0.05$	
$B^0 \rightarrow \rho^- l^+ \nu_l$	$2.40 \pm 0.24$	$2.34 \pm 0.28$
$B^0 \rightarrow \rho^- \tau^+ \nu_\tau$	$1.04 \pm 0.10$	
$B^+ \rightarrow \rho^0 l^+ \nu_l$	$1.29 \pm 0.13$	$1.07 \pm 0.13$
$B^+ \rightarrow \rho^0 \tau^+ \nu_\tau$	$0.56 \pm 0.06$	

tributions and relativistic transformation of the meson wave function from the rest to the moving reference frame. In Tables II, III we present results of the updated calculation of the semileptonic  $B \rightarrow \pi(\rho)l\nu_l$  decay branching fractions in comparison with experimental data [4, 10]. We also give predictions for the heavy-to-light semileptonic decays involving  $\tau$  lepton  $B \rightarrow \pi(\rho)\tau\nu_\tau$ . Corresponding plots for the semileptonic differential decay rates are shown in Fig. 2. The value of the CKM matrix element  $|V_{ub}|$  obtained from the comparison of our predictions and measured  $B \rightarrow \pi(\rho)l\nu_l$  decay rates amounts  $|V_{ub}| = (4.05 \pm 0.20) \times 10^{-3}$ . As we see from Tables II, III calculated decay branching fractions agree well with available experimental data.

TABLE III: Comparison of theoretical predictions and experimental averages for exclusive determinations of  $Br(\bar{B} \rightarrow \pi l \bar{\nu})$  ( $\times 10^{-4}$ ).

	$Br$	$Br(q^2 < 12 \text{ GeV}^2)$	$Br(q^2 < 16 \text{ GeV}^2)$	$Br(q^2 > 16 \text{ GeV}^2)$
Theory	$1.43 \pm 0.14$	$0.72 \pm 0.07$	$0.96 \pm 0.09$	$0.47 \pm 0.05$
Experiment [10]	$1.42 \pm 0.03 \pm 0.04$	$0.81 \pm 0.02 \pm 0.02$	$1.05 \pm 0.02 \pm 0.03$	$0.37 \pm 0.01 \pm 0.02$

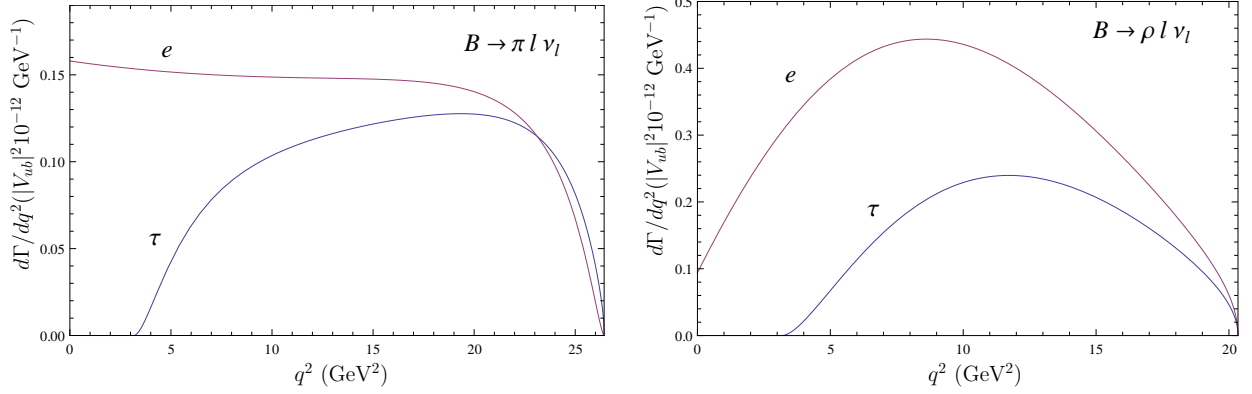


FIG. 2: Predictions for the differential decay rates of the  $B \rightarrow \pi(\rho)$  semileptonic decays.

TABLE IV: Ratios of branching fractions of the semileptonic  $B \rightarrow M\tau\nu_\tau$  and  $B \rightarrow Ml\nu_l$  ( $M = D^{(*)}, \pi, \rho$ ) decays.

Ratio	Theory	Experiment [3]
$R(D)$	0.315	$0.440 \pm 0.058 \pm 0.042$
$R(D^*)$	0.260	$0.332 \pm 0.024 \pm 0.018$
$R(\pi)$	0.63	
$R(\rho)$	0.43	

Next we consider the ratios of  $B$  decays involving light  $l$  and  $\tau$  leptons

$$R(M) = \frac{Br(B \rightarrow M\tau\nu_\tau)}{Br(B \rightarrow Ml\nu_l)}, \quad M = D^{(*)}, \pi, \rho. \quad (4)$$

In these ratios the corresponding CKM matrix elements cancel and as a result uncertainties are significantly reduced. We present our predictions for  $R(M)$  in Table IV and confront them with recent experimental values for  $R(D)$  and  $R(D^*)$  from BaBar Collaboration [3]. It is important to note that most of the above mentioned form factor uncertainties cancel out in these ratios. We see that our predictions for these ratios are lower than the experimental values by  $1.75\sigma$  for  $R(D)$  and by  $2.4\sigma$  for  $R(D^*)$ .

We use the obtained above value of the CKM matrix element  $|V_{ub}| = (4.05 \pm 0.20) \times 10^{-3}$  to calculate the branching fractions of the leptonic  $B \rightarrow L\nu_L$  ( $L = l, \tau$ ) decays:

$$Br(B \rightarrow L\nu_L) = \frac{G_F^2}{8\pi} M_B M_L^2 \left(1 - \frac{M_L^2}{M_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B, \quad (5)$$

where  $M_B$  and  $M_L$  are the  $B$  meson and  $L$  lepton masses,  $G_F$  is the Fermi constant,  $f_B$  is the decay constant and  $\tau_B$  is the life time of the  $B$  meson. The decay constants of light and heavy-light mesons were calculated in our model in Ref. [11] with the consistent account of relativistic effects including contributions of the negative-energy quark states. The obtained value of the  $B$  meson decay constant  $f_B = (189 \pm 9)$  MeV is in good agreement with the unquenched lattice QCD calculations  $f_B^{\text{lattice}} = (189 \pm 4)$  MeV [12]. The result for the branching fraction of the  $B \rightarrow \tau\nu_\tau$  decay is compared to the experimental data in Table V,

TABLE V: Calculated [ $f_B = (189 \pm 9)$  MeV,  $f_{B_c} = (433 \pm 5)$  MeV,  $|V_{ub}| = (4.05 \pm 0.20) \times 10^{-3}$ ,  $|V_{cb}| = 0.039 \pm 0.0015$ ] and measured branching fractions of the leptonic  $B$  and  $B_c$  decays ( $\times 10^{-4}$ ).

Decay	Theory	Experiment [4]	Experiment [5]	Experiment [6]
$B \rightarrow \tau \nu_\tau$	$1.04 \pm 0.15$	$1.65 \pm 0.34$	$1.54^{+0.38+0.29}_{-0.37-0.31}$	$1.83^{+0.53}_{-0.49} \pm 0.24$
$B \rightarrow \mu \nu_\mu$	$0.0046 \pm 0.0007$	$< 0.01$		
$B_c \rightarrow \tau \nu_\tau$	$178 \pm 22$			
$B_c \rightarrow \mu \nu_\mu$	$0.73 \pm 0.09$			

where the PDG [4] as well as new Belle [5] and BaBar [6] data are given. We find that our prediction for this decay branching fraction is lower than central experimental values. However, taking into account large uncertainties, it agrees well with separate Belle and BaBar measurements, but deviates by  $1.25\sigma$  from the averaged value [4].

In Table V we also give predictions for the leptonic decays of the  $B_c$  meson. For the evaluation we use the CKM matrix element  $|V_{cb}| = 0.039 \pm 0.0015$ , obtained above, and the value of the  $B_c$  decay constant  $f_{B_c} = (433 \pm 5)$  MeV calculated in our model [8]. This value is in good agreement with the recent lattice calculation  $f_{B_c}^{\text{lattice}} = 0.427(5)$  GeV [13]. We see that the predicted  $BR(B_c \rightarrow \tau \nu_\tau)$  is of order of few percent and, in principle, can be measured at LHC where  $B_c$  mesons are copiously produced. It is important to check this prediction experimentally.

In summary, we calculated branching fractions of the semileptonic and leptonic  $B$  meson decays in the framework of the relativistic quark model, paying particular attention to the decays involving  $\tau$  lepton. The obtained results are compared to the recent experimental data. We find that for the decay branching fractions reasonable agreement between the theory and experiment is observed for the CKM matrix elements  $|V_{cb}| = 0.039 \pm 0.0015$  and  $|V_{ub}| = (4.05 \pm 0.20) \times 10^{-3}$ . The largest deviations of our theoretical predictions from experimental data occur for the ratios  $R(D)$  and  $R(D^*)$ , and they are about  $1.75\sigma$  and  $2.4\sigma$ , respectively.

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